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PROSPECTIVE ELEMENTARY TEACHERS' SUSPENSION OF SENSE-MAKING WHEN SOLVING PROBLEMATIC WORD PROBLEMS

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This study investigates the extent to which pre-service elementary teachers (PETs) use their real-world knowledge to solve problems for which the result of the arithmetic operation is problematic, if one takes into consideration the reality of the context. A paper-and-pencil test was administered to 566 PETs enrolled in mathematics content courses. The test included 8 experimental items and 4 buffer items. The findings for a sample of 68 PETs are not very encouraging. The total number of realistic responses varied from 5 to 58 (out of 68 possible for each problem).

Word arithmetic word problems play an important role in learning mathematics at the elementary school level. There are several practical and theoretical reasons of the inclusion of arithmetic word problems in the elementary curriculum. First, they provide contexts in which students can use their mathematical knowledge so they can develop their problem-solving abilities, an important goal of learning mathematics. Second, word problems provide practice so students can develop their abilities to use their knowledge in real-life situations. Third, word problems serve as motivators so students can see the relevance of the procedures and algorithms learned in school. Fourth, word problems have the potential to provide students with rich contexts and realistic activities in which to ground mathematical concepts and, thus, facilitate the learning of more complex concepts. Finally, word problems provide students with experiences to learn how to use mathematical tools to model aspects of reality, that is, to describe, analyze, and predict the behavior of systems in the real world (Burkhardt, 1994; De Corte, Greer, & Verschaffel, 1996; Verschaffel, Greer, & De Corte, 2000; Verschaffel & De Corte, 1997).

Some critiques (e.g., Gerofsky, 1996; Lave, 1992; Nesher, 1980) argue, however, that the mathematics curriculum fails to achieve these lofty goals because traditional instructional tasks tend to focus on a straightforward application of procedures and computations to solve artificial problems unrelated to the real world. As a result, students tend to approach word problems, more often than desirable, in a superficial and mindless way with little, if any, disposition, to modeling and realistic interpretation. Several pieces of research provide empirical evidence to these claims

(Davis, 1989; De Corte & Verschaffel, 1989; Greer, 1993, 1997; Reusser, 1988; Reusser & Stebler, 1997; Schoenfeld, 1991; Silver, Shapiro, & Deutsch, 1993; Verschaffel, 1999; Verschaffel & De Corte, 1997; Verschaffel, De Corte, & Lasure, 1994).

Purpose of the Study

The purpose of the study was to examine prospective elementary teachers' (PETs) reactions and responses to problematic arithmetic word problems for which the solution is not the result of application of the most obvious arithmetic operation suggested by the two numbers given in the problem statement.

As suggested by the research literature, elementary school children tend to ignore the realistic constrains of the context embedded in the statement of the problem, a phenomenon that Schoenfeld (1991) coined "suspension of sense-making." Several critics and researchers argue that children' suspension of sense-making is the result of school practices (Davis, 1989; Greer, 1993; Nesher, 1980; Schoenfeld, 1991; Silver, Shapiro, & Deutsch, 1993). To develop children' disposition to realistic modeling, we must change curriculum and instructional tasks. Since the teacher has an important role in the construction or selection of learning tasks and opportunities, one may argue that researchers and curriculum developers need to understand teachers' reactions and responses to problematic problems.

Theoretical and Empirical Background

Mathematical modeling is the process of representing aspects of reality by mathematical means. In particular, the solution of some physical or real-world problems requires some form of mathematization. That is, the construction of a mathematical model. The complexity of the process of mathematization depends, of course, on the nature of the problem. There are several proposed models of representing reality by mathematical means (e.g., Silver, Shapiro, & Deutsch, 1993; Verschaffel, Greer, & De Corte, 2000), but Silver et al's model (Fig. 1) suffices for our purposes.

According to Silver, Shapiro, and Deutsch's model, a simplified version of the process of mathematical modeling consists of four different stages: understanding of the problem, construction of a model or selection of a mathematical procedure, the execution of the procedure, and the interpretation or evaluation of the outcomes of the procedure.

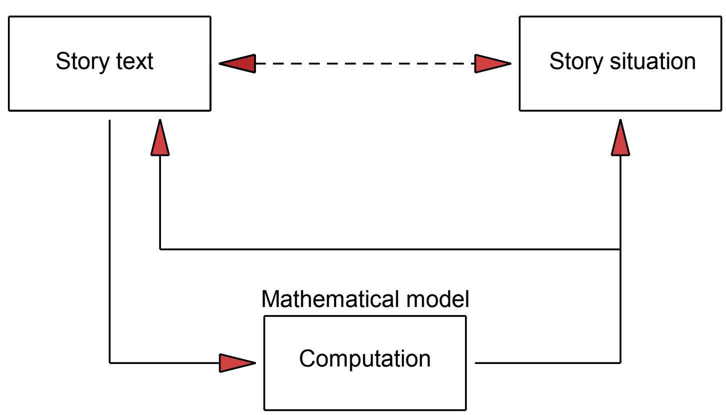

Fig. 1: Silver et al.'s (1993) referential-and-semantic-processing model for successful solutions

The first stage of the process of mathematical modeling involves understanding the problem

situation embedded in the story text. That is, we need to understand the given or known facts, the unknown information, the superfluous data, and missing information. The second phase involves the construction of a mathematical model or selection of a suitable procedure, operation, or algorithm whose outcome will lead us to the solution of the problem. To perform the second stage of the modeling process successfully, we must understand the mathematical structure of the problem. That is, we must understand the interconnections or relationships among the different types of information related to the solution of the word problem. The third stage of the problem involves mainly performing the computation, procedure, or algorithm either with paper and pencil or using a computational device. Finally, we should interpret and assess the outcome of the mathematical procedure in terms of the realistic context embedded in the story text of the word problem or in terms of the real-world story situation. It is during this step that we need to focus on the meaning of the result of the mathematical model so we can establish the connection between the outcome of the computation and the solution to the real-world story problem. It is during this stage that we need to assess whether our modeling assumptions are realistic or reasonable.

Silver, Shapiro, and Deutsch's model implies that there are three main potential sources of error when solving word problems: lack of understanding of the mathematical structure of the problem, which leads students to select an inappropriate procedure, executing the procedure incorrectly, and failing to interpret or assess the result of the procedure in terms of commonsense or everyday-life knowledge. Silver, Shapiro, and Deutsch (1993) examined 195 middle grade students' solution processes and their interpretation of solutions to the following problem:
The Clearview Little League is going to a Pirates game. There are 540 people, including players, coaches, and parents. They will travel by bus, and each bus holds 40 people. How many buses will they need to get to the game?

Their analysis revealed that 91% of the students selected an appropriate procedure (e.g., long division, repeated multiples, repeated additions, etc.), but only 61% of these students performed it flawlessly (about 56% of the total number of students). Overall, the researchers found that only 43% of the total number of students gave the correct answer (14) to the problem. However, some of these students provided inappropriate interpretations or justifications. For example, one student wrote "14 buses because there's leftover people and if you add a zero you will get 130 buses so you sort of had to estimate. Are we allowed to add zeros?" (p. 124-125). The researchers also reported that about 55% of the students did not get the correct answer because either they did not properly interpret the outcome of the computation or executed incorrectly the procedure. These computational mistakes could have been prevented if students had interpreted their solutions appropriately. Silver, Shapiro, and Deutsch proposed the model displayed in Figure 2 as a graphical representation of unsuccessful solutions that are due to a failure to connect the outcome of the procedure to the real-world context embedded in the story problem.

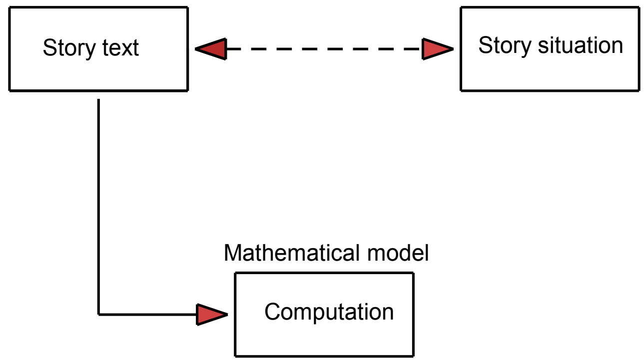

Figure 2: Silver et al.'s (1993) referential-and-semantic-processing model for unsuccessful solutions

Other pieces of research have amply documented elementary school children' improper modeling assumptions when solving arithmetic word problems. Some further examples of the word problems that students have been asked to solve are the following:

1) What will be the temperature of water in a container if you pour 1 liter of water at 80° and 1 liter of water of 40° into it? (Nesher, 1980)
2) John's best time to run 100 m is 17 sec. How long will it take to run 1 km? (Greer, 1993)
3) Lida is making muffins that require 3/8 of a cup of flour each. If she has 10 cups of flour, how many muffins can Lida make? (Contreras & Martínez, 2001)
4) In September 1995 the city's youth orchestra had its first concert. In what year will the orchestra have its fifth concert if it holds one concert every year? (Verschaffel, De Corte, & Vierstraete, 1999)

In their study with 75 fifth graders in Flanders, Verschaffel, De Corte, and Lasure (1994) reported that only 7 (9%) students provided a realistic and correct response to the temperature problem. Similarly, in the same study, these researchers found that only 2 (3%) responses included realistic answers or reactions to the running problem. In another study, Contreras and Martínez (2001) focused on prospective elementary teachers' solution processes and realistic reactions to the third problem. Their analysis revealed that only 19 (28%) of the participants' responses contained a realistic solution to the problem, but none of the participants made any comments about the problematic nature of the problem.

Verschaffel, De Corte, and Vierstraete (1999) addressed upper elementary school children' difficulties in modeling and solving nonstandard additive word problems involving ordinal numbers. The participants were administered a paper-and-pencil test consisting of 17 word problems, 9 of which were experimental items and 8 buffer items. The result of the straightforward arithmetic operation yields the correct answer for three of the nine experimental items. An example of such a problem is "In January 1995 a youth orchestra was set up in our city. In what year will the orchestra have its fifth anniversary? However, the solution of the remaining six experimental items is either 1 more or 1 less that the result of the straightforward arithmetic operation of the two given numbers. An example of such a problem is problem 4 stated above. Overall, the researchers found that the percentage of correct responses for each of the six problematic items was less that 25%. An error analysis revealed that 83% of the errors made on these problems were ± errors. In other words, most of the children' errors were due to their interpretation that the result of the addition or subtraction of the two given numbers yielded the correct answer.

Although research has convincingly documented elementary school children' strong

tendency to model problematic problem unrealistically, the generalizability of the findings to more mature students, such as prospective elementary teachers, has not been established empirically. On one hand, since PETs have had even more experiences with traditional school problems, we may argue that there is no reason to expect that prospective elementary teachers would use their real-world knowledge and realistic considerations in their solution processes of problematic word problems. On the other hand, we may claim that PETs may have faced real-world problem situations outside school more often than young children and, having a more developed mathematical knowledge, have a stronger disposition to activate their real-world knowledge when confronted with problematic problems whose realistic solutions require taking into consideration contextual real-world knowledge. In the present study we focus on the extent to which the findings from previous research with pupils are generalizable to prospective elementary teachers.

Methods and Sources of Evidence

The total sample of participants consists of 566 PETs enrolled in different sections of mathematics content courses for elementary teachers at a Southern University in the United States. The present paper reports the results of two groups (68 PETs) for which the analysis has been completed. The PETs had not been previously engaged in any intentional or systematic modeling activities or tasks.

A paper-and-pencil test consisting of 8 experimental items and 4 buffer items was administered to the PETs during regular class instruction. The 8 experimental items (Table 1) were problematic in the sense that the outcomes of the arithmetic operations performed with the given numbers in the problem story does not provide the answer to the problem, if one takes into consideration the real-world situation embedded in the contextual problem story. The buffer items, on the other hand, were standard routine problems whose solution is the straightforward result of the operation performed with the given numbers. The experimental items were adapted from Verschaffel and De Corte's (1997) study. An example of a buffer item is "Joel is building a collection of 175 different stamps. He has already collected 107 different stamps. How many more stamps does he need to complete the collection?"

Table 1: The eight experimental items

1. 1175 supporters must be bused to the soccer stadium. Each bus can hold 40 supporters. How many buses are needed? (Carpenter, Lindquist, Matthews & Silver, 1983).
2. 228 tourists want to enjoy a panoramic view from the top of a high building that can be accessed by elevator only. The building has only one elevator with a maximum capacity of 16 persons. How many times must the elevator ascend to get all the tourists on the top of the building? Verschaffel, 1995)
3. At the end of the school year, 50 elementary school children try to obtain their athletics diploma. To receive the athletic diploma they have to succeed in two tests: running 400 m in less than 2 minutes and jumping 0.5 m high. All the children participated in both tests. Nine children failed the running test and 12 failed the jumping test. How many children did not receive their diplomas? (Verschaffel, 1995)
4. Carl and George are classmates. Carl has 9 friends that he wants to invite to his birthday party. On the other side, George has 12 friends that he wants to invite to his birthday party. Since Carl and George have the same birthday, they decide to give a party together. They invite all of their friends. All their friends come to the party. How many friends are there at the party? (Nelissen, 1987)
5. A man wants to have a rope long enough to stretch between two poles 12 m apart, but he has only pieces of rope 1.5 m long. How many of these pieces would he need to tie together to stretch between the poles? (Greer, 1993)
6. Steve has bought 12 planks of 2.5m each. How many 1 m planks can he saw out of these planks? (Kaalen, 1992)
7. Sven's best time to swim the 50 m breaststroke is 54 seconds. How long will it take him to swim the 200 m breaststroke? (Greer, 1993)
8. The flask is being filled from a tap at a constant rate. If the water is 4 cm deep after 10 seconds, how deep will it be after 30 seconds? (This problem was accompanied by a picture of a cone-shaped flask) (Greer, 1993)

After each problem, we have indicated its original source; however, in some cases the numbers were replaced by others.

Students' written responses to the problems were the source of data. Written directions asked students to show all their work to support each of their answers and to write down any questions or concerns they may have about each problem**.** We recognize that written responses have some intrinsic limitations when compared to verbal protocols. However, written protocols allow researchers to collect data from large samples. Moreover, some researchers (Hall, Kibler, Wenger, & Truxaw, 1989) have validated the use of written responses to infer cognitive processes.

Analysis and Results

Each response to problems 1 and 2 was coded as correct or incorrect. Each response to problems 3-8 was coded as correct, partially correct, or incorrect. Two independent raters judged every response. A response was judged as correct if it included a realistic numerical answer that estimated or indicated the range of possible solutions and took into account the contextual restraints of the real-world problem situation. A response was judged partially correct if it was incomplete or wrong but included a realistic comment suggesting that the student displayed awareness of the contextual restraints of the real-world problem situation. A response was judged incorrect when it did not suggest any awareness of the contextual restraints of the real-world problem situation. The inter-rater agreement was 99.6%. Table 2 summarizes the results of the analysis.

Table 2: The number and percentage of correct, partially correct, and correct responses for the 8 experimental items.

Problem	Number and percent of correct responses	Number and percent of partially correct responses	Number and percent of incorrect responses
1	55 (81%)	0 (0%)	13 (19%)
2	58 (85.5%)	0 (0%)	10 (14.5%)
3	3 (4.5%)	13 (19%)	52 (76.5%)
4	3 (4.5%	15 (22%)	50 (73.5%)
5	2 (3%)	4 (6%)	62 (91%)
6	14 (20.5%)	1 (1.5%)	53 (78%)
7	1 (1.5%)	4 (6%)	63 (92.5%)
8	0 (0%)	5 (7.5%)	63 (92.5%)

As shown in Table 2, PETs' performance on most items was alarmingly poor: The percentage of incorrect responses ranged from a high 92.5% for items 7 and 8 to 14.5% for item 2. Overall, the percentage of realistic responses (correct responses and partially correct responses) on the 8 problematic items was only 33%. We should notice, however, that the number of realistic responses was considerable greater for the division problems involving remainders, problems 1 and 2). If we exclude these two problems from our analysis, then the percentage of realistic responses for the remaining 6 problems is only 16%.

Discussion and Conclusion

The purpose of the present study was to collect systematically empirical data about the extent to which prospective elementary teachers activate their real-world knowledge when solving problems whose solution in not the direct result of an arithmetic operation. Using similar problems and methodology as previous studies (e.g., Verschaffel & De Corte, 1997; Verschaffel, De Corte, & Lasure, 1994), a test consisting of 8 problematic items and 4 standard problems was administered to a sample of 566 PETs. The analysis has been completed for 68 PETs (2 groups) and it is reported in the present article.

Although previous studies have convincingly demonstrated children' strong tendency to ignore the contextual realities embedded in the story of the problem situation, we were hoping that our findings with prospective elementary teachers would be much more encouraging. After all, prospective elementary teachers are part of a more mature and experienced population and it is reasonable to assume that they have an understanding of the contextual knowledge needed to realistically solve the problems. Therefore, the question of PETs' failure to activate this knowledge needs to be further discussed and investigated. We offer several tentative hypotheses to explain PETs' lack of disposition to model contextual word problems realistically.

First, children and PETs' lack of activation of their real-world knowledge may be due to their constant exposure to traditional and stereotypical school word problems. If this is the case, then this tendency may remain constant or get stronger with additional years of immersion in the mathematical culture of traditional classrooms. Future research is needed to better understand the effects of traditional learning environments on students', including PETs, failure to activate their real-world knowledge to solve problematic problems.

A second possible explanation to understand PETs' tendency to ignore the contextual realities of the situation embedded in the problem story is that they lack enough world-real knowledge of the situational context of the problematic problems. Even though this seems unlikely, follow-up studies should provide empirical data to confirm or refute this hypothesis.

A third explanation may be that PETs approached the problematic problems in an automatic way thinking that they were standard mathematical problems without reflecting on the contextual realities of the problem. Further research is needed to better understand PETs' suspension of sense-making when solving these types of problems.

In conclusion, this study provides, at the very least, some empirical evidence that PETs lack an initial disposition or reaction to consider the contextual restraints of problems grounded in the real world. Further research is needed to better understand PETs' apparent suspension of sense-making when engaged in solving problems that require realistic interpretations.

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